Review Chapter 19

$\vec{y} = (y_1, y_2, \cdots, y_n)$ — y_1: q; $y_2, \cdots, y_n$: $-x$

$q \le f(x) \Rightarrow$ ①单调 ②凹函数 $\Leftrightarrow$ Y是凸集.

1. The technological constraints of the firm are described by the production set, which depicts all the technologically feasible combinations of inputs and outputs, and by the production function, which gives the maximum amount of output associated with a given amount of the inputs.

2. Another way to describe the technological constraints facing a firm is through the use of isoquants—curves that indicate all the combinations of inputs capable of producing a given level of output.

3. We generally assume that isoquants are convex and monotonic, just like well–behaved preferences.

Review Chapter 19 (Cont'd)

1. The marginal product measures the extra output per extra unit of an input, holding all other inputs fixed. We typically assume that the marginal product of an input diminishes as we use more and more of that input.

2. The technical rate of substitution (TRS) measures the slope of an iso-quant. We generally assume that the TRS diminishes as we move out along an isoquant—which is another way of saying that the isoquant has a convex shape.

3. Returns to scale refers to the way that output changes as we change the scale of production. If we scale all inputs up by some amount t and output goes up by the same factor, then we have constant returns to scale. If output scales up by more that t, we have increasing returns to scale; and if it scales up by less than t, we have decreasing returns to scale.

Intermediate Microeconomics

Chapter 20: Profit Maximization

Instructor: Ziyang Chen

Econ Department, Business School, Nanjing University

Economic Profit

A firm uses inputs j = 1,…,m to make products i = 1,…,n.

Output levels are $y_1,\ldots, y_n$.

Input levels are $x_1,\ldots, x_m$.

Product prices are $p_1,\ldots, p_n$.

Input prices are $w_1,\ldots, w_m$.

The economic profit generated by the production plan $(x_1,\ldots, x_m, y_1,\ldots, y_n)$ is

会计利润 $\pi = p \cdot y - w \cdot x$

$$\prod = p_1 y_1 + \cdots + p_n y_n - w_1 x_1 - \cdots - w_m x_m$$

ISO-profit Curve

A $\$\prod$ iso−profit line (等利润线) contains all the production plans that yield a profit level of $\$\prod$.

The equation of a $\$\prod$ iso−profit line is

$$\prod = \mathrm{py} - \mathrm{w}_1\mathrm{x}_1 - \mathrm{w}_2\widetilde{\mathrm{x}_2}$$

i.e.,

$$\mathrm{y} = \frac{\mathrm{w}_1}{\mathrm{p}}\mathrm{x}_1 + \frac{\prod + \mathrm{w}_2\widetilde{\mathrm{x}_2}}{p}$$

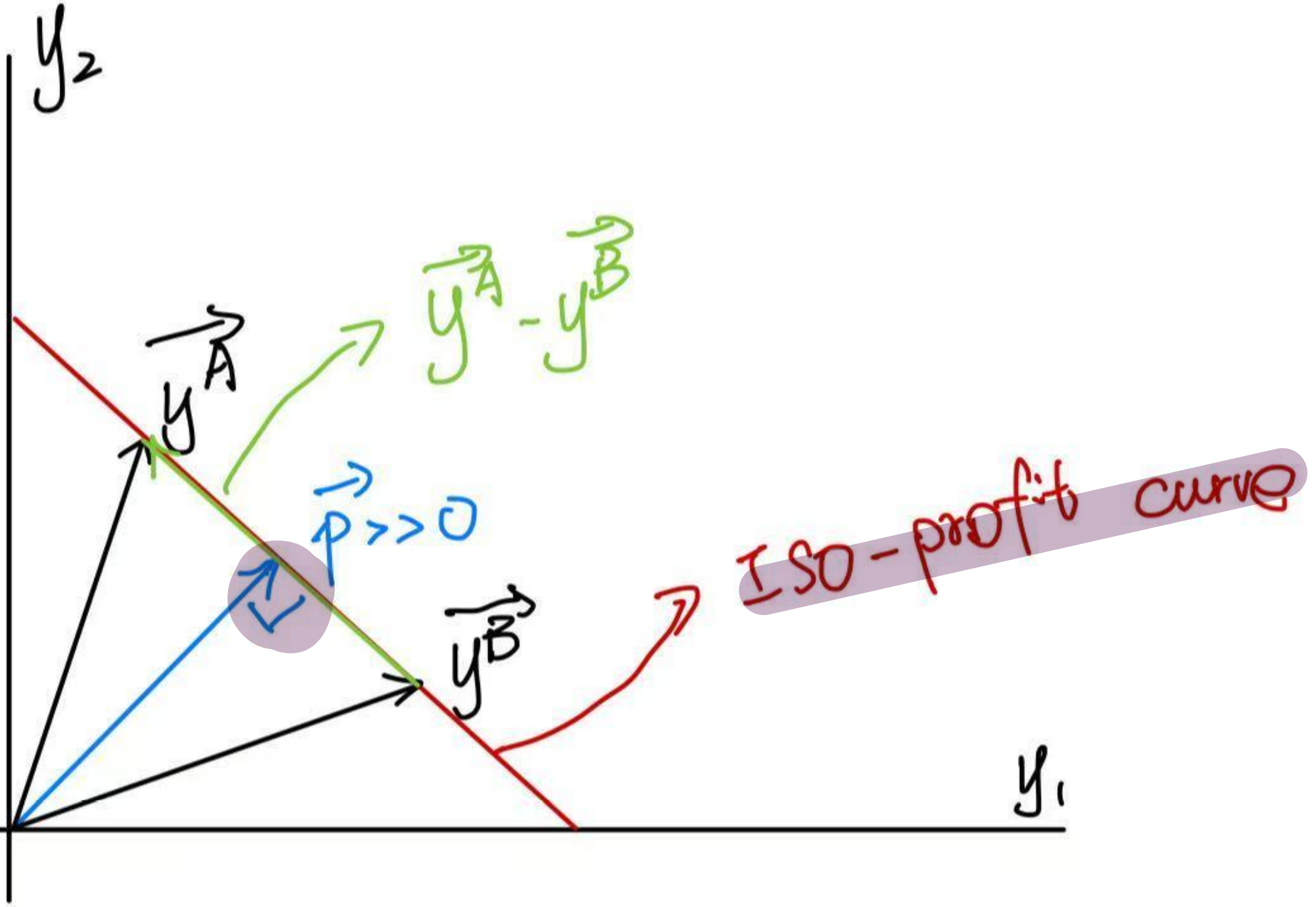
ISO-profit Curve

$$\{y: \bar{\pi} \equiv \vec{p}\,\vec{y}\}$$

$$\vec{p}\,\vec{y^A} = \vec{p}\,\vec{y^B} = \bar{\pi}$$

$$\vec{p}\,(\vec{y^A} - \vec{y^B}) = 0$$

$\Rightarrow$



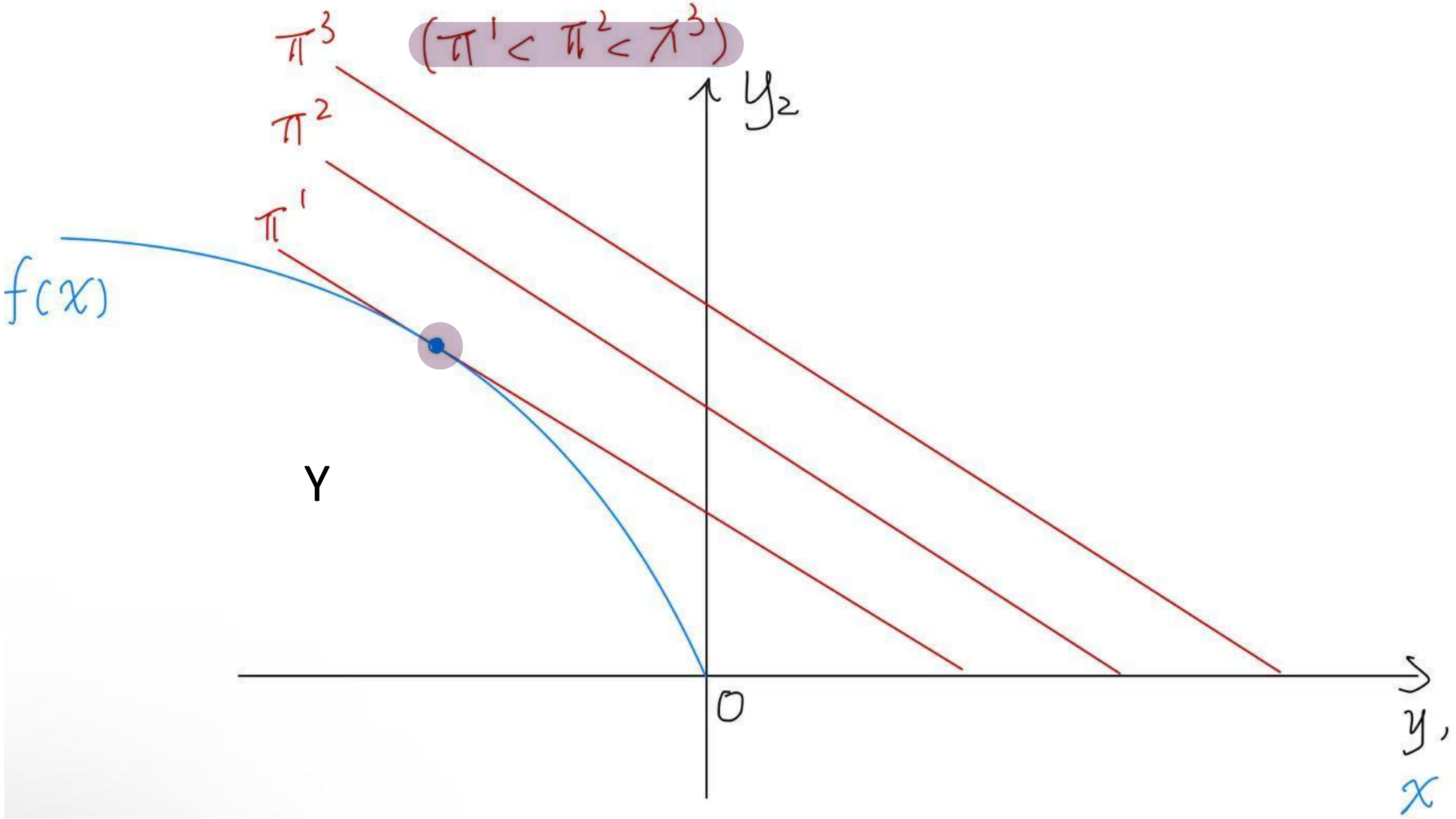
Profit Maximization

The firm’s problem is to locate the production plan that attains the highest possible iso-profit line, given the firm’s constraint on choices of production plans.

Q: What is this constraint?

A: The production function

Profit Maximization



Profit Maximization

$\max p \cdot y. \quad y \in Y$

$Y = \{(q, -x) : f(x) \geq q\}$

$p = (p, w)$

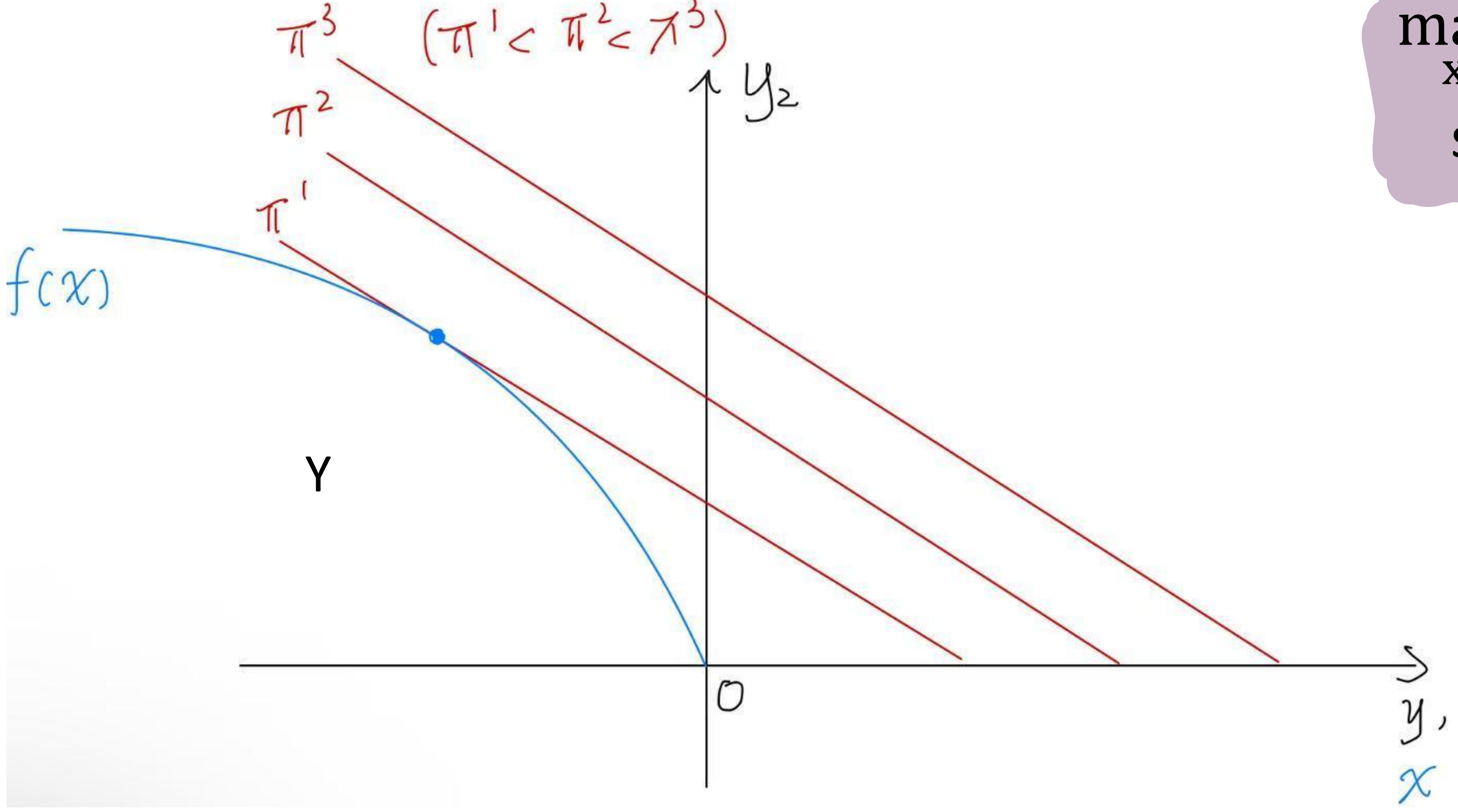
$\Rightarrow \max_{q,-x} pq - wx$

s.t. $f(x) \geq q$

$\Downarrow$

$\max_x pf(x) - wx$

s.t. $f(x) = q$



$$\max_x pf(x) - wx$$

$$\text{s.t, } f(x) = q$$

Profit Maximization

$y = x_1^{\frac{1}{3}} \tilde{x}_2^{\frac{1}{3}}$ (x_2 fixed)

p_1, p_2, w

$MP = \frac{\partial y}{\partial x_1} = \frac{1}{3} x_1^{-\frac{2}{3}} \tilde{x}_2^{\frac{1}{3}}$

FOC. $MP_1 \cdot p = w$

$\frac{1}{3} x_1^{-\frac{2}{3}} \tilde{x}_2^{\frac{1}{3}} p = w$

$\left(\frac{3w}{p} \frac{1}{\tilde{x}_2^{\frac{1}{3}}}\right)^{-\frac{3}{2}}$

$\Rightarrow x_1^* =$

Suppose one input and one output. The profit-maximization problem is

$$\max_x pf(x) - wx$$

FOC is

$$0 = pf'(x) - w \Rightarrow pf'(x) = w$$

$f'(x) = MP$

Rearrange to

$$\frac{w}{p} = MP = f'(x) \Rightarrow x^* = ?, y^* = ?$$

$MP \times p$ is the marginal revenue product(边际收益产量) of input 1, the rate at which revenue increases with the amount used of input 1.

If $MP \times p > w$, then profit increases with x.

If $MP \times p < w$, then profit decreases with x.

Profit Maximization

Now allow the firm to vary both input levels, i.e., both x_1 and x_2 are variable. The profit-maximization problem is

长期

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

FOCs are

$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_1} - w_1 = 0$$

$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_2} - w_2 = 0$$

Demand for inputs 1 and 2 can be solved as

$$x_1 = x_1(w_1, w_2, p)$$

$$x_2 = x_2(w_1, w_2, p)$$

Profit Maximization: An Exercise

The production function is

$$y = x_1^{1/3} x_2^{1/3}$$

$$\max_{x_1, x_2} \; p f(x_1, x_2) - w_1 x_1 - w_2 x_2.$$

FOCs: $p \frac{\partial f}{\partial x_1} - w_1 = 0 \Rightarrow \frac{1}{3} p x_1^{-\frac{2}{3}} x_2^{\frac{1}{3}} = w_1$

$p \frac{\partial f}{\partial x_2} - w_2 = 0 \quad \frac{1}{3} p x_1^{\frac{1}{3}} x_2^{-\frac{2}{3}} = w_2$

(1) Write down the profit-maximization problem.

(2) Solve for the demand for inputs 1 and 2.

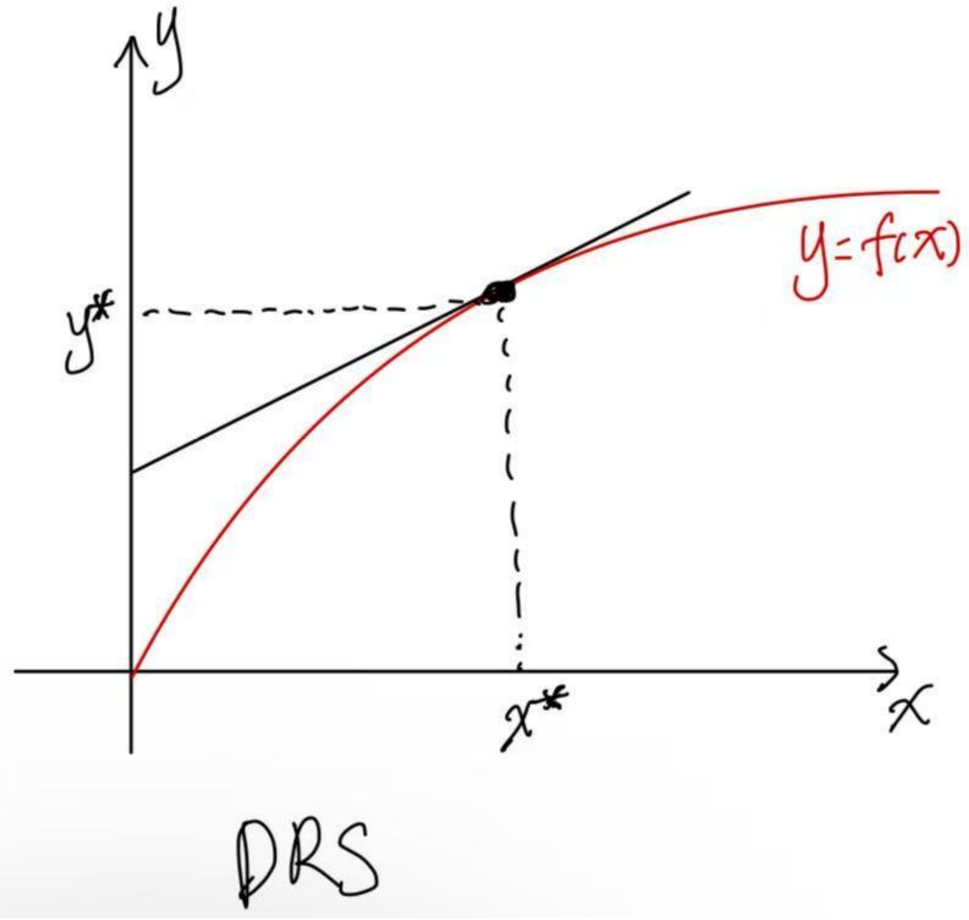
(3) $y^* =?$

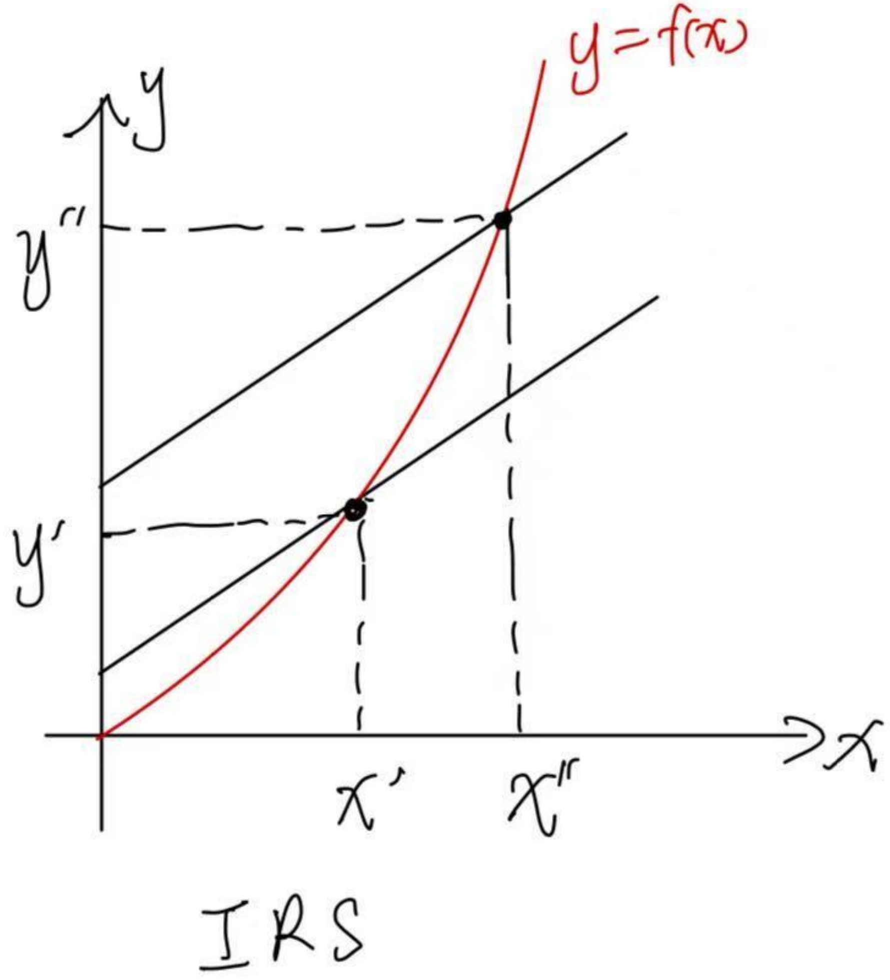
Returns-to Scale and Profit Maximization

If a competitive firm's technology exhibits decreasing returns-to-scale then the firm has a single long-run profit-maximizing production plan.

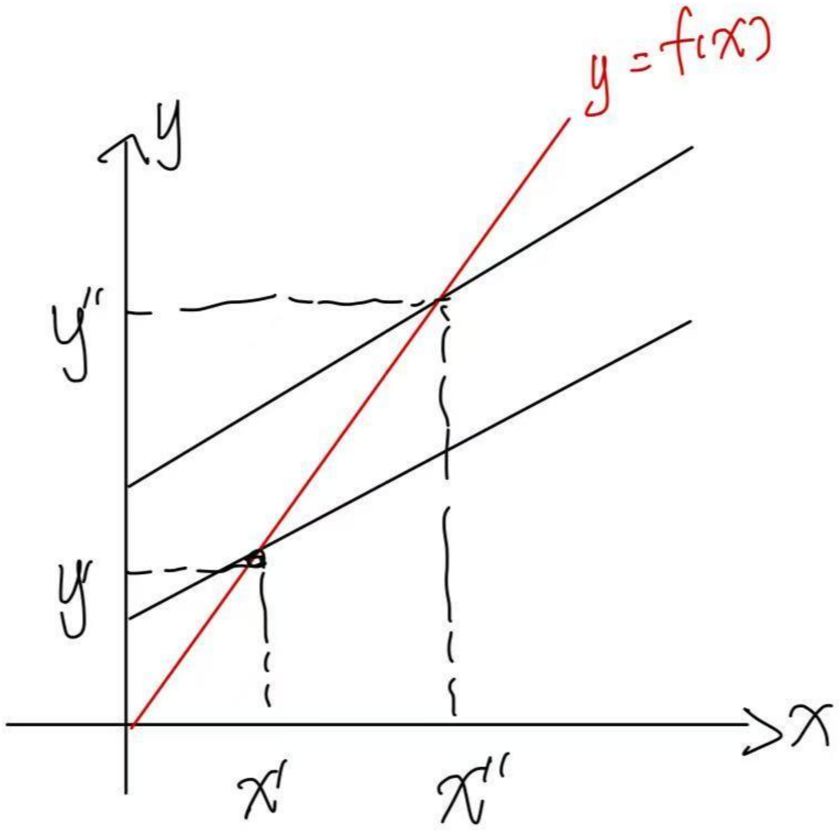
What if the competitive firm's technology exhibits constant returns-to-scale?

Returns-to Scale and Profit Maximization





Returns-to Scale and Profit Maximization



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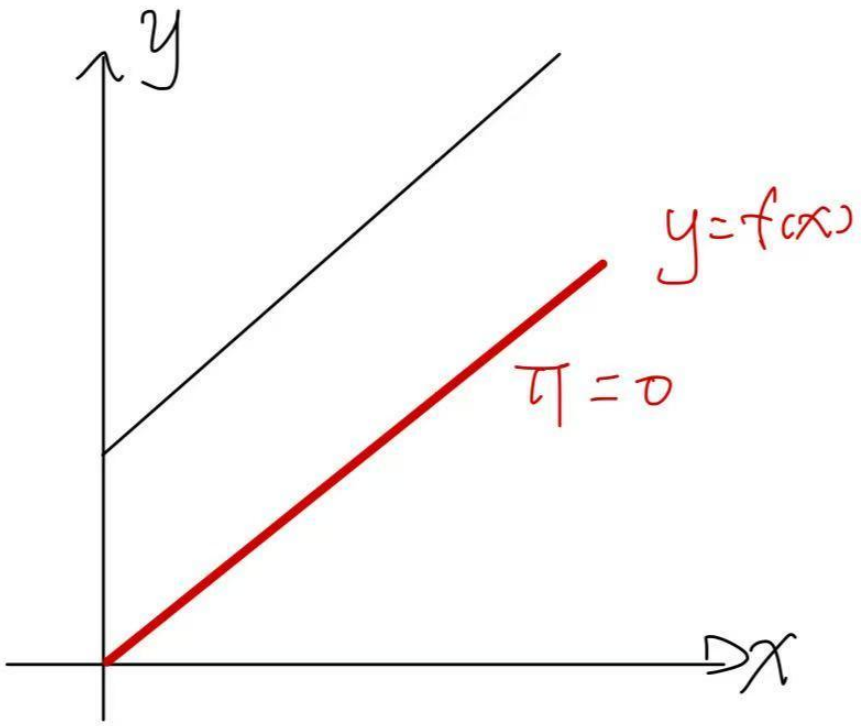


CRS II.

Revealed Profitability

$p_1: y_1$ $\quad p_1 y_1 \geq p_1 y_2$

$p_2: y_2$ $\quad p_2 y_2 \geq p_2 y_1$

$\Rightarrow (p_1 - p_2)(y_1 - y_2) \geq 0$ $\quad p_1 y_1 + p_2 y_2 \geq p_2 y_1 + p_1 y_2$

supply rule

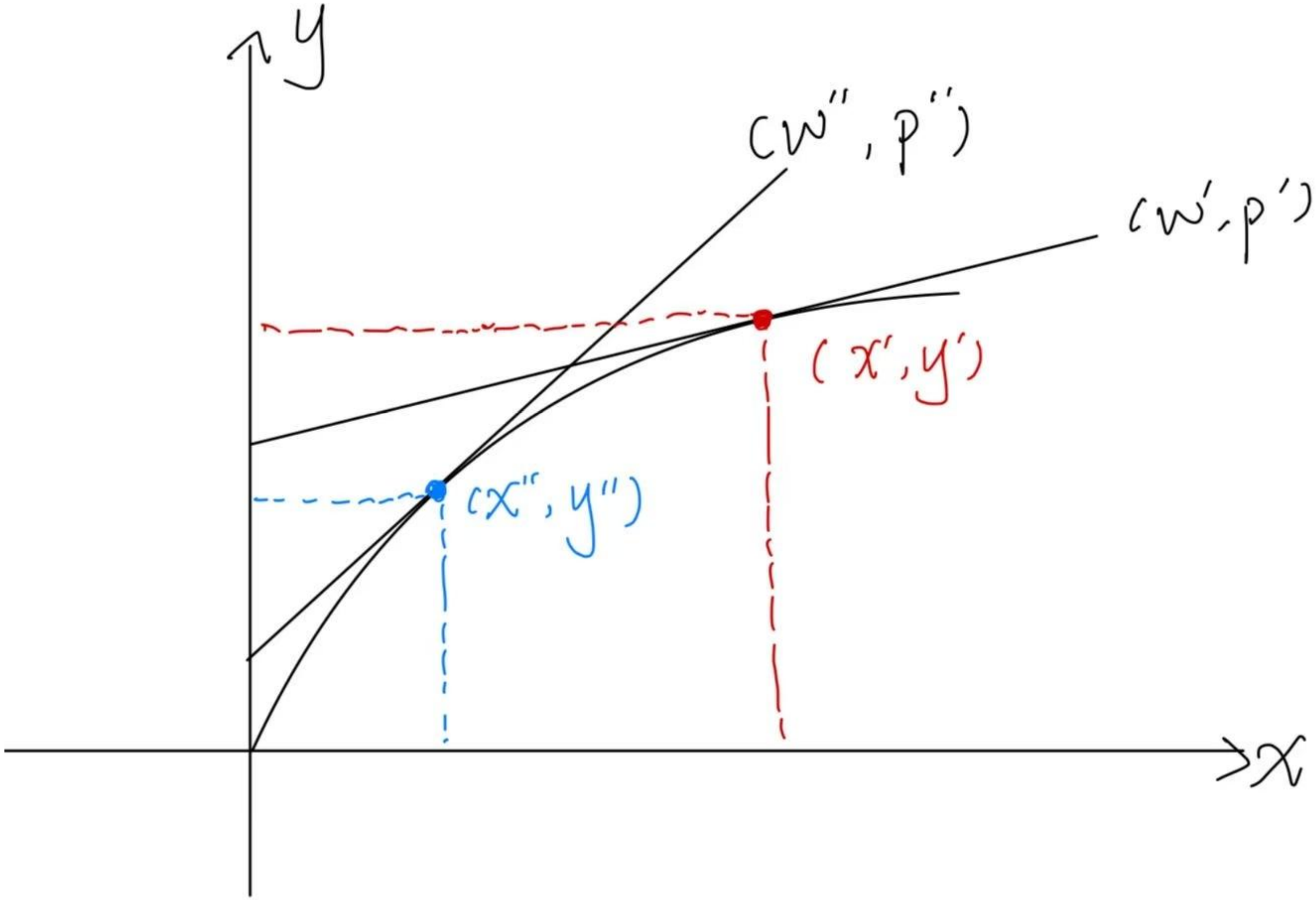
Consider a competitive firm with a technology that exhibits decreasing returns-to-scale.

For a variety of output and input prices we observe the firm’s choices of production plans.

What can we learn from our observations?

If a production plan (x’,y’) is chosen at prices (w’,p’) we deduce that the plan (x’,y’) is revealed to be profit-maximizing for the prices (w’,p’).

Revealed Profitability



Profit Maximization: An Exercise

The production function is

$$y = x_1^{\alpha} x_2^{1-\alpha}$$

Prices: p, w_1, w_2. Solve the profit-maximization problem.

$$\max_{x_1, x_2} \; py - w_1 x_1 - w_2 x_2$$

FOCs. $\alpha p x_1^{\alpha-1} x_2^{1-\alpha} - w_1 = 0$

$(1-\alpha) p x_1^{\alpha} x_2^{-\alpha} - w_2 = 0$

$\Rightarrow \frac{w_1}{w_2} = \frac{\alpha}{1-\alpha} \frac{x_2}{x_1}$

$\left(\frac{x_1}{x_2}\right)^{\alpha-1} = \frac{w_1}{\alpha p}$

$\left(\frac{x_1}{x_2}\right)^{\alpha} = \frac{w_2}{(1-\alpha) p}$

$\frac{x_1}{x_2} = \left(\frac{w_2}{(1-\alpha)p}\right)^{\frac{1}{\alpha}}$

$x_1 = x_2 \left(\frac{w_1}{\alpha p}\right)^{\frac{1}{\alpha-1}}$

$(1-\alpha) p \, x_2^{\alpha} \left(\frac{w_1}{\alpha p}\right)^{\frac{\alpha}{\alpha-1}} = w_2$

$x_2^{\alpha} = \frac{w_2}{(1-\alpha)p} \left(\frac{w_1}{\alpha p}\right)^{\frac{\alpha-1}{\alpha}}$

$x_2 = \left(\frac{w_2}{(1-\alpha)p} \left(\frac{w_1}{\alpha p}\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{1}{\alpha}}$

$\pi = py - w_1 x_1 - w_2 x_2$

$= \left[p\left(\frac{1-\alpha}{\alpha}\frac{w_1}{w_2}\right)^{1-\alpha} - w_1 - \left(\frac{1-\alpha}{\alpha} w_1\right)\right] x_1$

$= \left[p\left(\frac{1-\alpha}{\alpha}\frac{w_1}{w_2}\right)^{1-\alpha} - \frac{1}{\alpha} w_1\right] x_1$

>0. x_1越多越好

$=0$. $x_1 = ?$

<0. $x_1 = 0$

Profit Maximization: An Exercise

The production function is

$$y = x_1^{\alpha} x_2^{1-\alpha}$$

Prices: p, w_1, w_2. Solve the profit-maximization problem.

FOCs imply: $\frac{w_1}{w_2} = \frac{\alpha}{1-\alpha}\frac{x_2}{x_1}$

$$\Rightarrow \; p \; \begin{matrix} < \\ = \\ > \end{matrix} \; \frac{w_1^{\alpha} w_2^{1-\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$$